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UNDER
PRESSURE
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20
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60
80
100

ball, mass, balance, pump, pressure, ideal gas, elastic collision, coefficient of restitution

physics, mathematics, ICT

This unit can be used to teach students of different ages from primary, middle and high school. Both parts can be adapted to different levels:
Level 1: For primary school (age: 9–12 years)
Level 2: For secondary school (middle school, age: 12–15 years)
Level 3: For secondary school (high school, age: 15–18 years)

1 | SUMMARY

Did you ever wonder how important the pressure of the air inside the football is? This unit presents different activities focusing on this pressure. The first activity starts with the measurement of the mass of the air inside the ball and emphasises its direct proportionality with the inside pressure. The second activity studies the dependence of the maximum height reached by the ball after the first collision or bounce on the air pressure inside the ball and at the same time shows the importance of the surface condition of the ground.

2 | CONCEPTUAL INTRODUCTION

Our goal is to emphasise that with simple experiments the students can measure the mass of the air inside the ball and then verify the linear dependence between the pressure and the mass according to the ideal gas law. Finally, they will study the importance of pressure in the process of bouncing and will apply the law of the conservation of mechanical energy.

2 | 1 Part 1: Air mass versus pressure

See the details of the activities in part 3 *What the students do*.

Level 1:

Two different and independent activities can be carried out. The first one focuses on the mass of the air and how to measure the mass of the air inside the ball. The teacher could use an inquiry-based approach by asking students “how could you find the mass of air inside a ball?”. The students will propose and conduct experiments, such as using a balance, inflating the ball and checking the mass of the ball when it is inflated. In the second activity, the students will focus on the volume and on the methods of finding the volume of the ball (e.g. with a bucket of water).

Level 2:

Measure the mass of the air inside the ball at different pressures. Find the link between the pressure and the mass of the air (Assumption: the volume of the ball does not change when the pressure increases). The students can draw a graph of the mass of the gas versus the pressure. The students can also measure the volume of the ball. This experiment can also be used to discover the buoyancy of the ball (in air).

Level 3:

The students can do the same experiments as those of level 2. They will compare their graph of the dependence between the mass and the pressure of the air inside the ball with the ideal gas law and they will calculate different gas parameters on the basis of the slope of the graph.

2 | 2 Part 2: Bouncing height vs. pressure

Level 1:

Focus on differences between the heights (qualitatively): Drop two balls from the same height and note the direct effect of different pressures within the ball. Choose a procedure, choose the data you will collect, collect the data, and discuss them after the experiment is finished.

Level 2:

Focus on differences between the heights (qualitatively): Measure the maximum height after the first bounce, then repeat the experiment ten times, looking for a way to detect the height, for example by making a high-speed film with a smartphone. Learn about random and other factors that account for different results, and calculate the average height.

Level 3:

Focus on using a mathematical model of free fall to analyse the data. Starting with level 2, analyse the data in order to find the loss of energy by using the formula $E_{pot} = m \cdot g \cdot h$ and comparing the energy at the beginning of the experiment (h = 1 m or another value) and after the ball’s first contact with the ground. The students can also calculate the time of a bounce and the maximum speed of the first contact with the ground, and then try to measure it. Finally, they can compare the potential energy and the kinetic energy (E_{pot} and E_{kin}) and calculate the coefficient of restitution (see *3.2.1*).

E_{pot}: potential energy [J]
m: mass of the ball [g]
g: gravitational acceleration; $g = 9.81 \frac{m}{s^2} = 9.81 \frac{N}{kg}$
h: height attainted by the ball [m]

Part 2 can be done on different surfaces, such as grass, the gym floor, asphalt, concrete, wet grass, short and higher grass and finally sand. Students on all levels should state their hypotheses, discuss them and analyse the experiments on different levels. Going further, it could be interesting to develop a table that shows the pressure that is necessary in order to get the same height of bounce on different surfaces, for instance in different stadiums.

3 | WHAT THE STUDENTS DO

This unit is divided into two parts: measuring the mass of the gas versus the pressure inside the ball, and measuring the dependence between the height of the bounce and the pressure inside the ball.

There are two different ways of measuring the pressure.

The relative pressure is the difference between the pressure inside the ball and the atmospheric pressure (outside the ball); a manometer is used to measure the relative pressure. We use this pressure in part 1.

The absolute pressure is the total amount of the pressure. We use this pressure in part 2.

3|1 Part 1: Measure the mass of gas vs. the pressure

Equipment needed: a pump, a manometer (pressure measurement system), a balance (with an accuracy of 0.1 g and a range of measurement between 0 and 1,000 g), a nozzle to inflate the ball, a glass to put the ball on the balance, a football.

If the school does not have this equipment, the experiment can be done with cheap devices.

(The easiest way is to have the manometer on the pump. If this is not the case, it is easy to find a cheap manometer for car tires; the nozzle is the same as the one used for a ball.)

3|1|1 Procedure

We describe all the details of our suggested procedure here. Some parts of it may be left out if they do not fit the level of your group of students.

FIG. 1 Ball inside the bucket

- **Measure the volume of the ball (with and without air inside)**
  To measure the volume of the ball, you can use a bucket filled with water and measure the different levels of water with and without the ball. Be careful, because the football skin is made of leather and it could absorb water, which would increase the mass of the ball. To avoid this phenomenon, you can put the ball in a plastic bag. The pressure of the water around the ball will "stick" the bag against the ball. The volume will be the same with and without a bag.

FIG. 2 Measure the level to get the volume of water

  If you make the measurement without a plastic bag around the ball, do it after the mass measurement.

  The volume can be measured with different levels of the water inside the bucket. If the students cannot calculate the volume of the water in the bucket, they can fill the bucket completely, push the ball inside, and measure the volume of the water that overflows.

  In this case the volume of the empty ball is 1.65 L and the volume of the full ball is 5 L. This means that 5 L – 1.65 L = 3.35 L of air is inside the ball.

- **Measure the mass with air inside**
  Put the glass on the balance, tare the balance, put the ball on the balance, measure the mass.

  In this experiment we use a balance with an accuracy of 0.1 g (between 0 and 1,000 g), a football and a pump with a manometer.

FIG. 3 The ball on the balance

FIG. 4 Measure the mass of the empty ball

- **Measure the mass of the ball without any air inside**
(for instance $m_{ball} = 408.0$ g)

- **Inflate the ball to get the same pressure inside and outside the ball**
The relative pressure, or the difference between the pressure inside and outside the ball, is $P = 0$ bar. Measure the mass of the ball $m_{ball} = 408.0$ g (The same mass as before!).

3 | 1 | 2 Analysis: Why is the mass the same both with and without air inside the ball?

- **Hint:** The air around us is a fluid and it creates a force that has the same properties as the force created when we put something into water.
- **Answer:** The mass of the air inside the ball is balanced by the buoyancy of the air around the ball.
- Measure the mass of the same ball with a different pressure. The manometer will give the relative pressure.
- Collect the data in a spreadsheet. For instance, you can measure the mass for a relative pressure $P = 0.35$ bar; $P = 0.5$ bar; $P = 0.6$ bar; $P = 0.75$ bar; $P = 0.9$ bar; $P = 1.05$ bar, or choose other pressures.
- Draw the curve m vs P.
- Find the best fit of the curve (it is a linear function).
- Find the link between the slope of the straight line and the ideal gas law: $P \cdot V = n \cdot R \cdot T$

To help the students understand the ideal gas law, the teacher can give a few hints.

- **First hint:** The linear curve has the formula
$m_{total} = a \cdot P + m_{ball}$
or $m_{total} = m_{gas} + m_{ball}$.

It means that: $m_{gas} = a \cdot P$.

- **Second hint:** $n_{gas} = \frac{m_{gas}}{M_{gas}}$.

m: mass [g]
P: relative pressure [Pa]
a: coefficient of the slope of the curve $[\frac{g}{bar}]$
V: volume [m^3]
n: amount of substance [mol]
M: molar mass $[\frac{g}{mol}]$
R: ideal gas constant, $R = 8.31 \frac{J}{K \cdot mol}$
T: temperature [K]

- **Third hint:** The gas (air) is roughly composed of 20 % of oxygen and 80 % of nitrogen.
$M_{O_2} = 32 \frac{g}{mol}$ and $M_{N_2} = 28 \frac{g}{mol}$

3 | 2 Part 2: Measure the bouncing height vs. the pressure

3 | 2 | 1 Theory

Have you ever wondered how important the inside air pressure is for a ball? We will show that the coefficient of restitution e (elasticity) depends on this pressure.

What is the coefficient of restitution? When a ball is falling, it lands at a certain velocity with respect to the ground, which is called its velocity of approach. After the elastic collision with the ground, the velocity of separation will have a value that is different from the velocity of approach because a part of the initial kinetic energy will be lost:

$$e = \frac{v_{separation}}{v_{approach}}.$$

It is very easy to calculate this coefficient if you measure the initial height h_1 from which the ball is falling and then measure the maximum height h_2 that can be reached after the ball bounces off the ground.

We use the law of the conservation of energy:

$$mgh_1 = \frac{mv^2_{approach}}{2} \qquad mgh_2 = \frac{mv^2_{separation}}{2}.$$

So: $e = \sqrt{\frac{h_2}{h_1}}$.

e: coefficient of restitution
v: velocity $[\frac{m}{s}]$
m: mass [g]
g: gravitational acceleration; $g = 9.8 \frac{m}{s^2} = 9.8 \frac{N}{kg}$
h: height [m]

3 | 2 | 2 **The experiment**

We drop the ball from a height (h_1) and then we note the height (h_2) of the ball's bounce after it hits the ground. We can measure these heights in the videos.

FIG. 5 Hold the ball at the height h_1 (left); drop the ball (right)

The experiment can be done with different kinds of balls and different kinds of surface [1].

4 | CONCLUSION

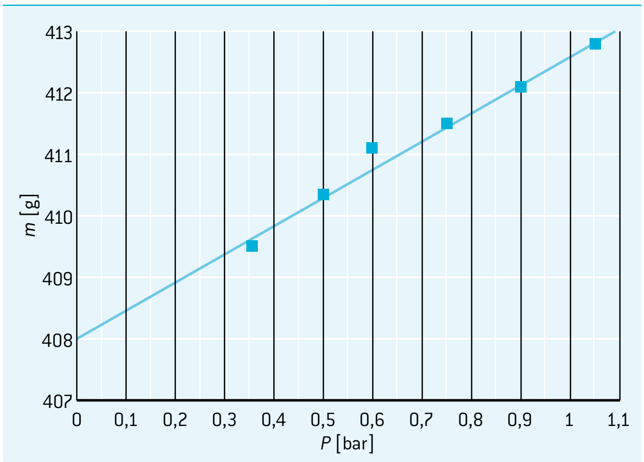
4 | 1 Part 1: Measure the mass of gas versus the pressure

4 | 1 | 1 Example of measuring the mass vs. the pressure of a ball

The mass of the ball is $m_{ball} = 408.0$ g at $P = 0$ bar.
The volume of the air in the ball is $V = 3.35$ L.

FIG. 6 m [g] vs. P [bar] (relative pressure)

P [bar]	m [g]
0.75	411.5
0.35	409.5
1.05	412.8
0.9	412.1
0.6	411.1
0.5	410.3



4 | 1 | 2 **Example of calculation with the ideal gas law:**

Here the formula of the curve is $m = 4.5711\,\frac{\text{g}}{\text{bar}} \cdot P + 408.0$ g.

We see that the value 408 is the mass of the empty ball in grams.

or $m_{total} = a \cdot P + m_{ball}$.
m: total mass [g]
P: pressure [bar]
a: coefficient of the slope of the curve $[\frac{\text{g}}{\text{bar}}]$

In this case $a = 4.5711\frac{\text{g}}{\text{bar}}$.

The value of a can be found by means of the law of ideal gases:
$P \cdot V = n \cdot R \cdot T$.
P: pressure [Pa], 1 bar = 10^5 Pa
V: volume [m^3]
n: amount of gas [mol]
R: ideal gas constant, $R = 8.31\,\frac{\text{J}}{\text{K} \cdot \text{mol}}$
T: temperature [K]
M: molar mass $[\frac{\text{g}}{\text{mol}}]$

This means that $n_{gas} = \frac{P \cdot V}{R \cdot T}$ and $m_{gas} = M_{gas} \cdot \frac{P \cdot V}{R \cdot T}$

or $m_{gas} = \frac{M_{gas} \cdot V}{R \cdot T} \cdot \text{P}$

and we have already seen in *3.2.1* that $m_{gas} = a \cdot P$,

so $a = \frac{M_{gas} \cdot V}{R \cdot T}$.

The air is roughly composed of 20 % of oxygen and 80 % of nitrogen, so here

$$M_{gas} = \frac{20 \cdot M_{O_2} + 80 \cdot M_{N_2}}{100}$$

$$M_{gas} = \frac{20 \cdot 32\frac{\text{g}}{\text{mol}} + 80 \cdot 28\frac{\text{g}}{\text{mol}}}{100}$$

$$M_{gas} = 28.8\,\frac{\text{g}}{\text{mol}}.$$

With this ball
$V = 3.35\text{ L} = 3.35 \cdot 10^{-3}\,\text{m}^3$
$T = 20\,°\text{C} = 293$ K

$$a = \frac{M_{gas} \cdot V}{R \cdot T}$$

$$a = \frac{28.8\frac{\text{g}}{\text{mol}} \cdot 3.35 \cdot 10^{-3}\,\text{m}^3}{8.31\frac{\text{J}}{\text{K mol}} \cdot 293\text{ K}} = 3.96 \cdot 10^{-5}\frac{\text{g}}{\text{Pa}}.$$

This is the value when P is measured in Pa. For P in bar, the value has to be multiplied by 10^5 (because 1 bar = 10^5 Pa).
$a = 3.96\,\frac{\text{g}}{\text{bar}}$

The best fit of the curve is $a = 4.57\,\frac{\text{g}}{\text{bar}}$.

If we compare the two results, the relative deviation between the two results is:

$$d = \frac{4.57 - 3.96}{4.57} = 0.13.$$

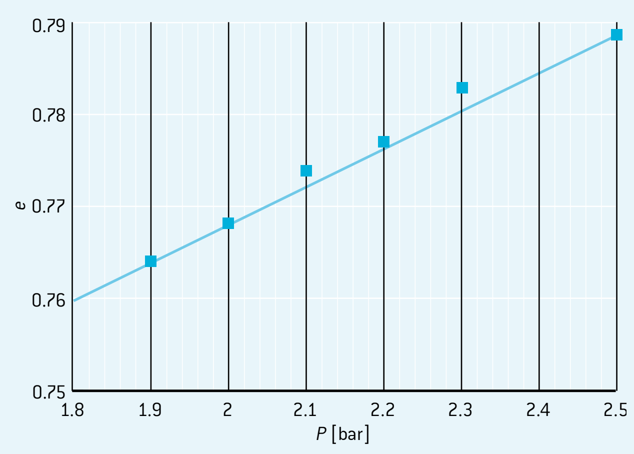
We can discuss the errors linked with the measure: Here the accuracy of the manometer is 0.05 bar on a measure of about 1 bar. There can still be air inside the ball when we measure the volume of the empty ball.

4 | 2 Part 2: Measure the bounce vs. the pressure

In our experiment we changed the inside air pressure in two different balls and we obtained the following results:

FIG. 7 Coefficient of restitution *e* vs. absolute pressure *P* (Ball 1)

P [bar]	*e*
1.9	0.764
2.0	0.768
2.1	0.774
2.2	0.777
2.3	0.783
2.5	0.789



Here P is the absolute pressure in bar.

For the first ball the dependence is linear because the pressure variation is not so large.

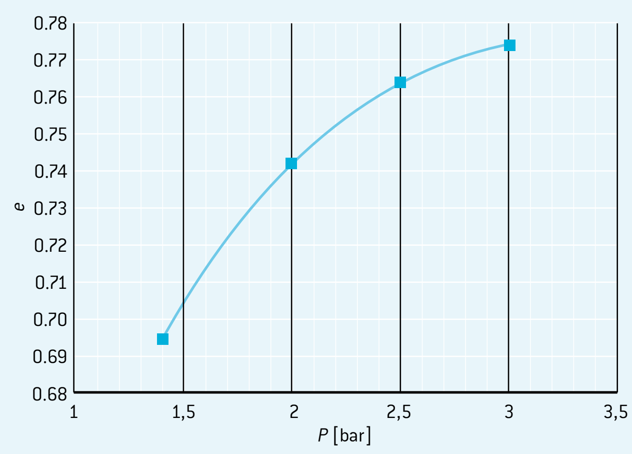
For the second ball we obtained a curve. When the pressure is too large the ball loses its elasticity and the coefficient of restitution seems to reach a limit.

In these two experiments the ball was dropped down on a floor, and you can see that the coefficient of restitution is about 0.77 for a pressure of 3 bar.

Then we changed the surface, but the inside air pressure remained 3 bar. On grass the coefficient of restitution was smaller: $e = 0.57$. On synthetic grass the coefficient became 0.74 [1].

FIG. 8 Coefficient of restitution *e* vs. absolute pressure *P* (Ball 2)

P [bar]	*e*
1.4	0.695
2.0	0.742
2.5	0.764
3.0	0.774



5 | CONCLUSION

Footballs are very good tools for studying the laws of gases, the property of pressure and the efficiency of the bounces. The students can study the laws of physics by means of a ball, which is a piece of sports equipment. They can see the links between the laws of physics, in this case the law of ideal gases, and daily life.

It is also interesting to see that the activities of this unit can be taught to students at different age levels, ranging from 6 to 18 years. It is easy to fit these activities into any kind of curriculum.

6 | COOPERATION OPTIONS

We can share our results for different experiments with footballs.

To share your results, download the file and follow the instructions [1].

We are sure that the students can share their ideas about the differences between their measurements or their experimental devices. They can imagine other experiments with the ball: for example, filming the deformation of the ball during its collision with the ground and the influence of pressure on this process.

REFERENCES

[1] www.science-on-stage.de/iStage3_materials

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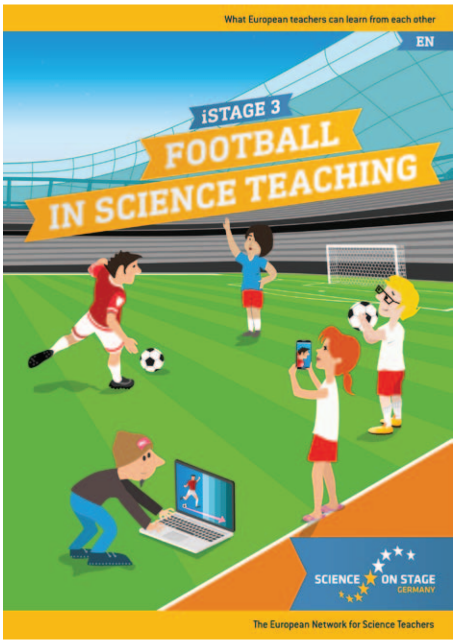

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