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C

Space Travel

INTRODUCTION

Imagine travelling from one planet to another. Why is it that we have to first travel in circles instead of taking the straight path? Before going on our trip, we must consider: the revolution velocity of our starting planet, the required spaceship velocity, and the optimum momentum for launching the spaceship (because if we miss it, we'll travel past the target planet, without even noticing it). Finally, we need to know the fuel economy of the trip (after all, we don't have gas stations in space). In this teaching unit, students study how a spaceship arrives on a circular orbit around a planet, and how it travels from one planet to another on a Hohmann transfer orbit. This unit is recommended for students aged 12 to 19 years. The applied subjects are: Physics, Mathematics, Informatics and Biology.

RESOURCES

The students need the following resources: computer Intel Dual Core with 2GB ram, 3D accelerated graphic card; Windows, Mac OSX, or Linux operating system; display resolution: min. 1024x768; installed software: Oracle Java JRE 1.6; license model: LGPL, Internet access.

For this teaching unit, we created two Java software applications: "Orbiting and Escaping" and "Solar System Travel" (see www.science-on-stage.de).

CORE

We will revise Newton's universal attraction law, circular motion quantities, Kepler's laws, and the potential and kinetic energy of a gravitational field.

Circular motion around a planet and escaping the planet's influence

The students should become familiar with the value of physical characteristics of circular motion of a satellite around a planet, or orbital motion of a planet. They should pay close attention to the velocity of circular trajectory around a planet, and the velocity required for escaping the gravitational field of this planet. They can find the formulae for these two velocities using the software "Orbiting and Escaping". They can verify the values with the software "Solar System Travel".

The application "Orbiting and Escaping" is based on the so-called "Newton's mountain model". Isaac Newton formulated a hypothetical experiment: If we climbed to the top of the highest mountain on Earth and from there, launched a projectile horizontally at appropriate velocity, during a time when Earth's atmosphere did not exist, we would have turned this projectile into an artificial satellite moving in circular orbit around Earth.

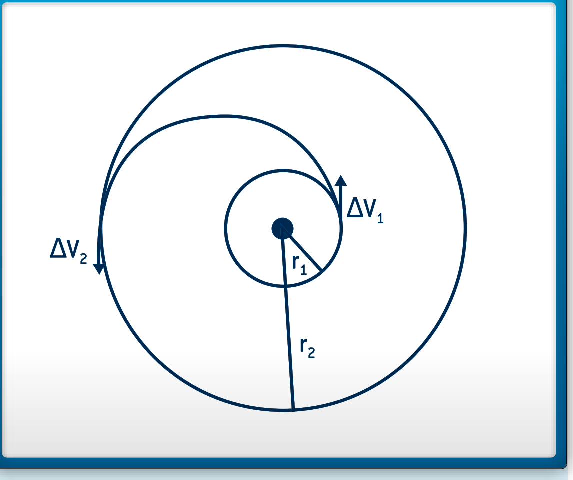
Travelling from one planet to the other on a Hohmann transfer orbit

Using the application "Solar System Travel", students have to make a choice and decide from which planet to which other planet they want to travel. By clicking on the Hohmann button they will be able to see the transfer ellipse between the planets. The ellipse shifts its position with the rotation of the start planet. It waits for the right time, when the planets' positions make travel possible. The application shows the spaceship travelling between the planets and calculates the time it needs to get to its destination.

The Hohmann transfer can be achieved with small thrusts initiated at the beginning and end of travel only. On the ellipse, the fuel consumption is set to minimum because this is where the changes of kinetic energy are smallest.

To travel from an orbit with the radius r_1 to another orbit with the radius r_2, we use an elliptical trajectory with the major axis = r_1+r_2, called Hohmann transfer orbit ①.

① Hohmann-Trajectory



The spaceship has to change its velocity twice, once at the beginning of the elliptical trajectory, and once at the end. This is done using the so-called velocity impulse delta-v (Δv). This change in velocity is a measure of the „effort" that is needed to change the trajectory when performing an orbital manoeuvre.

It is assumed that the spacecraft is moving on the initial circular orbit of the radius r_1 with velocity v_1, and on the final circular orbit of the radius r_2 with the velocity v_2. The gravitational force is equal to the centrifugal force:

$\frac{GMm}{r^2} + \frac{mv^2}{r}$, where M is the sun's mass, m is the spacecraft's mass and G is the gravitational constant. The velocity v_1 and v_2 are given by:

$$v_1 = \sqrt{\frac{GM}{r_1}} \quad \text{and } v_2 = \sqrt{\frac{GM}{r_2}} .$$

The transfer consists of a velocity impulse Δv_1, which propels the spaceship into an elliptical transfer orbit, and another velocity impulse, Δv_2, which propels the spacecraft into the circular orbit with radius r_2 and the velocity v_2. The total energy of the spacecraft is the sum of kinetic and potential energy. It is equal to half the potential energy at the semi-major axis a:

$$\frac{mv^2}{2} - \frac{GMm}{r} = \frac{GMm}{2a}, \text{ where } a = \frac{r1 + r2}{2}.$$

The solution for this equation yields the velocity at the initial point of the elliptical trajectory (perihelion) v'_1, and the velocity at the final point of the elliptical trajectory (aphelion) v'_2:

$$v'_1 = \sqrt{GM\left(\frac{2}{r_1} - \frac{2}{r_1+r_2}\right)} = v_1\sqrt{\frac{2r_2}{r_1+r_2}}$$

$$\text{and } v'_2 = \sqrt{GM\left(\frac{2}{r_2} - \frac{2}{r_1+r_2}\right)} = v_2\sqrt{\frac{2r_1}{r_1+r_2}} .$$

In this case, the changes in velocities are:

$$\Delta v_1 = v'_1 - v_1 = v_1\left(\sqrt{\frac{2r_2}{r_1+r_2}} - 1\right)$$

$$\text{and } \Delta v_2 = v_2 - v'_2 = v_2\left(1 - \sqrt{\frac{2r_1}{r_1+r_2}}\right).$$

Important

- If $\Delta v_i > 0$, the spacecraft burst is for acceleration. If $\Delta vi < 0$, the spacecraft burst is for deceleration.
- Kepler's third law yields the **transfer time** from the perihelion to aphelion:

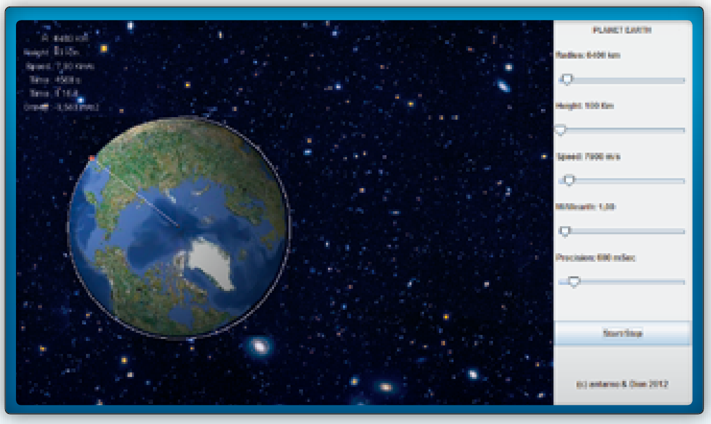
$$t = \pi\sqrt{\frac{(r1 + r2)^3}{8GM}} .$$

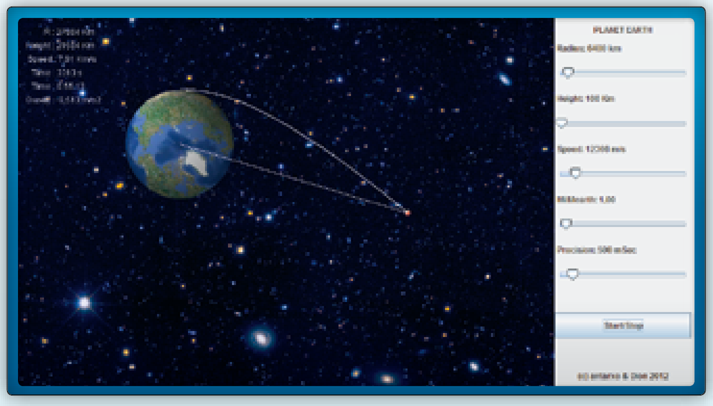
- **Waiting for the right moment**

The configuration of the two planets in their orbits is crucial. The destination planet and the spaceship must arrive in their respective orbits around the sun at the same point and simultaneously. This requirement for alignment gives rise to the concept of launch windows.

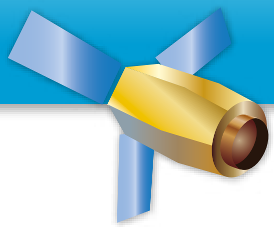
Student activities using the application "Orbiting and Escaping"

How to find the first and second cosmic velocities. The students can determine the circular velocity around the earth (1st cosmic velocity) and the escape velocity (the 2nd cosmic velocity) with the option "Earth" on the applet. They can see what happens when the initial velocity is greater or smaller than the 1st cosmic velocity.





How to define two formulae using this application. Using an essential experimental method, the students will determine the formulae that describe the circular velocity of a satellite orbit around a celestial body, and the escape velocity of this body. In doing so, they will discover the specific aspects of the Newtonian universal gravitation theory. At the basic level, when collecting and processing application data students will find each formula as proportionality. A more advanced approach allows them to define the coefficient of this proportionality changing it to equality.



With the option "Green Planet" (every other adjustment except Mi/M_{Earth} = 1 and radius = 6400 km, where M_i is the mass of the planet, expressed as the masses of Earth) the students may define the formula for the circular trajectory velocity. For this purpose, they choose a value for the planet's radius and enter the circular orbit velocity for different values of the planet's mass. When they reach a conclusion about the dependence between circular velocity and the planet's mass, they have to use these findings and transform them into a formula for proportionality. Repeating the same steps for a fixed value of the planet's mass and varying values for R (radius + height), students will arrive at a second proportionality.

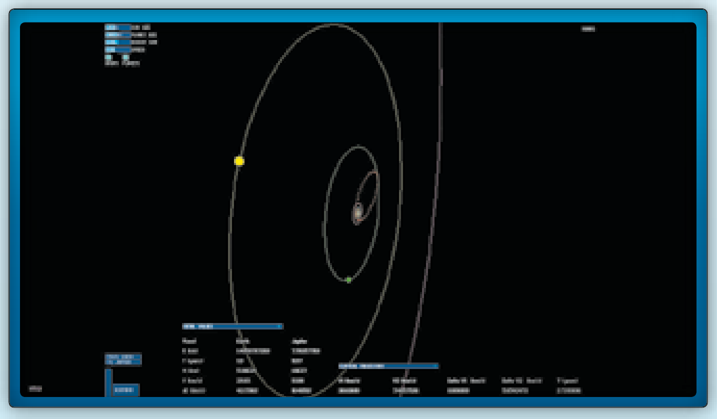
The process for finding the formula of the circular velocity around a planet will be completed when the students have changed the proportionality to equality. At first, they will merge the two proportionalities into one. Then they will draw a graph $v_2 = f(M_i/R)$ (where M_i is calculated in kg, with the $M_{Earth} = 6 \cdot 10^{24}$ kg). The slope of the graph gives the coefficient, which helps the students to find the equality.

Applying the same ideas, and following the same steps from the previous activity, the students can define the formula that describes the escape velocity, v_{escape}.

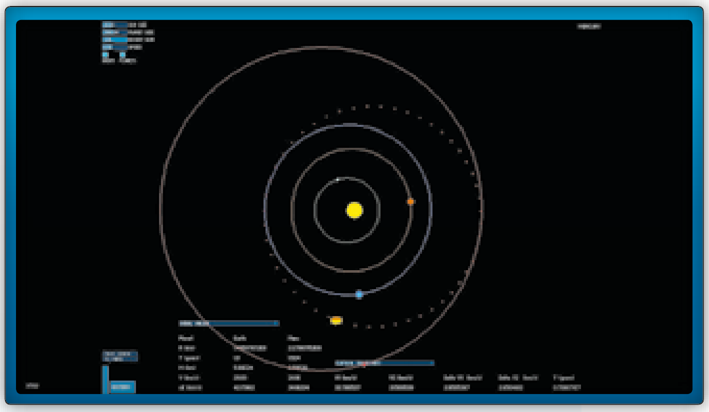
Student activities using the application "Solar System Travel"

Using the application, students can choose a journey between two planets. They can read the values for the initial velocities of every planet and for the Hohmann trajectory, and verify them with the newly created formulae from the first applet.

They can change the angle of the orbits using the SHIFT key, and zoom in and out with their mouse's SCROLL button.



The elliptical Hohmann trajectory (dotted) follows a rotational movement following the start planet of the spaceship. The students click the HOHMANN button and wait until the ellipse stops. At that moment the spacecraft begins its journey because the planets' configuration is favourable.



Studying orbital velocities and orbital periods for different planets

The students can conclude that the velocities of the planets are decreasing and the orbital periods are increasing with the increasing of the orbital radius. They can plot graphs for the planets' velocity and period evolution by increasing the orbital radius r, v = f(r) and T = f(r).

Comparison between different necessary velocity impulses (delta-v)

The students have to choose a Hohmann transfer orbit from Earth to Venus or Mercury. They can observe that $\Delta v_i < 0$. If they travel to one of the other planets, further away from the sun, they will observe that $\Delta v_i > 0$. They may conclude that when we intend to travel from a small orbit to a bigger one, the spacecraft has to accelerate and vice-versa: When we intend to travel from a bigger orbit to a smaller one, the spacecraft has to decelerate. The fuel consumption is the same.

Delta-v velocities versus escape velocities v_e

If the students enter in a table the delta-v values for each journey and the escape velocity ve for each planet, they may observe that in some cases the two values are very close. For example, it is impossible to go from the Earth to Uranus on a Hohmann orbit, so alternative solutions should be found.

Possible damage to astronauts' bodies

Using the application, students have to compare the transfer time t for different journeys. They can see that the required travel time is much longer when considering the appropriate "launch window". In this case, they have to consider the physiological consequences of prolonged space travel in microgravity (for example, weakening of the bones and straining heart muscles) under X and Gamma radiation (damage to cells), and in conditions of longitudinal acceleration (overconcentration of the blood in the head or feet of the astronauts). The students should research the biological damages of space travel and prepare posters on this topic.

CONCLUSION

While pursuing these simulations, students will be able to enrich and compare their knowledge base about the Solar System and space travel. This will broaden their horizon and they will become aware of the different problems of space travel. As we already pointed out, it is an interdisciplinary concept, involving not only Physics and Informatics but also Biology and Mathematics.

To build on this subject, students may also want to learn about possible perturbations during this kind of travel such as: third body perturbation, perturbation from atmospheric drag, and perturbation from solar radiation. They may want to try using other orbital manoeuvres as gravitational slingshot and Oberth effects.